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Finding Projection in the Two-Stage Supply Chain in DEA-R With Random Data Using (CRA) Model

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Abstract

Data envelopment analysis based on mathematical programming for decision-making units determines the efficiency score in addition to the projection of inefficient DMUs on the efficient frontier. Centralized Allocation Resource (CRA) with a two-stage linear programming model captures the projection of DMUs on the efficient frontier. But since the input and output vectors of each DMU in the DEA are crucial, they may be random data that follow a particular distribution. Hence, many applied studies face random data. This paper shows a two-stage supply chain with random data and the CRA model with ratio data has been used to calculate the projection of DMUs. In the end, the supply chain of 11 Iranian Airlines with random data during the period of 2011-2017 was considered concerning sustainability factors.

Keywords: Data envelopment analysis, Centralized Resource Allocation (CRA), DEA-R, Random data.

1 | Introduction

In 1957, Farrell [1] was the first to determine performance by nonparametric methods. The CCR model was introduced in 1978 by converting multiple inputs and outputs of a single decision unit into a virtual input and a virtual output. The BCC model is a special case of the CCR model introduced [2]. This model considered the conditions of variable return to scale for decision units.

Korhonen and Siitari [3] have used lexicographic parametric programming to identify efficient units in the DEA. We can refer to Korhonen and Siitari [4] for the identification of efficient units of large problems in DEA using the technique of dividing them into several smaller problems and the lexicographic parametric programming. Also, the presentation and interpretation of the appropriate pattern in DEA can be found in Korhonen.

The supply chain consists of a network of people, organizations, resources, activities, and technologies involved in the production and sales of a product. A supply chain can lead to lower costs, increased work speed, and increased company profitability. The cross-efficiency DEA approach was applied in the supply chain using capacity and demand information sharing [5]. Also, Chen and

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Yan [6] utilized centralized, semi-centralized, and hybrid DEA network models to evaluate the supply chain. Then, Seuring [7] reviewed articles on sustainable supply chains from 1998 to 2013, based largely on social factors. Then, Izadikhah and Saen [8] used a two-stage DEA model to evaluate supply chain negative data for 9 pasta factories in Iran. The sustainability of the two-stage supply chain with an optimistic and pessimistic frontier was studied by Badiezadeh et al. [9]. Bal and Satoglu [10] applied DEA models for the evaluation of the reverse supply chain by considering economic, social, environmental, and legal factors. Also, Zhang et al. [11] used a two-stage DEA model to reduce the environmental pollution of the two-stage supply chain in 30 provinces of China during the periods of 2011-2015.

The allocation efficiency was first introduced by Lozano and Villa [12]. They called this method the centralized allocation or intra-organization allocation since they assumed that all DMUs are under the control of centralized DM. In this method, with the lowest cost of total inputs, the best total output can be produced. Also, in evaluating a supply chain using the centralized resource allocation method, all DMUs are evaluated by one model, unlike the classic DEA models offered for each DMU individually. Asmild et al. [13] provided the allocative efficiency for the BCC model. Then, the allocative efficiency with interval data for five commercial banks in Malaysia was studied by Malekmohammadi et al. [14]. Following that, Hosseinzadeh Lotfi et al. [15] suggested the allocative efficiency model with random data for evaluating DMUs. Fang and Li [16] also calculated cost and revenue efficiency using the allocative efficiency for 20 fast-food restaurants in Hefei, China.

In various DEA models, inputs and outputs are assumed to be deterministic to evaluate the efficiency of DMUs. In other words, the DMUs under evaluation produce a certain amount of output with a given input value, but in reality, in many of the units under evaluation we cannot determine the amounts of inputs and outputs, so the inputs and outputs are uncertain and variables are random that may follow a particular statistical distribution. Therefore, in evaluating the supply chain using the centralized resource allocation method, when data are not certain across different factors, random data that follow a particular statistical distribution can solve this problem.

Hu et al. [17] calculated the cost efficiency of random data for 66 tourist hotels in Taiwan during 1997-2006. Azadi and Saen [18] applied Russell's model for random data with undesirable output for 20 IT companies in Iran. Then, Jin et al. [19] presented a model for assessing Asia-Pacific environmental efficiency with random data in 2010. Dong et al. [20] calculated the cost efficiency of random data for 41 Chinese banks during 1994-2007. Also, for evaluating 9 tomatoes paste production plants in Iran, Izadikhah and Saen [21] applied CCR fractional model to evaluate the two-stage sustainable supply chain with undesirable and random data. Liu et al. [22] conducted the calculation of the optimistic and optimistic efficiency of random data using DEA models. Finally, the envelopment form of DEA models with random fuzzy data was used to evaluate 20 affiliated companies to Iran Khodro Manufacturing Company in Iran by Tavassoli et al. [23].

Despić et al. [24] proposed DEA-R models in 2007 by combining data envelopment analysis and Ratio analysis. In general, if the input and output data of the decision-making units are ratio, the following two cases can be considered.

- *The input or output parameters of the decision-making units are inherently relative but the nominator and denominator of those ratios are clear [25] and [26].*
- *The input or output parameters of the decision-making units are inherently ratios, but the nominator and denominators of those ratios are not known [27] and [28].*

2 | Basic Concepts

This section describes two models: stochastic centralized resource allocation model and DEA-R input oriented:

2.1 | Stochastic Centralized Resource Allocation Model

Suppose $\tilde{x}_j = (\tilde{x}_{1j}, \tilde{x}_{2j}, \dots, \tilde{x}_{mj})^T$ and $\tilde{y}_j = (\tilde{y}_{1j}, \tilde{y}_{2j}, \dots, \tilde{y}_{sj})^T$ are the stochastic input and output vectors computed by the normal distribution and $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})^T$ and $y_j = (y_{1j}, y_{2j}, \dots, y_{sj})^T$ are the mean input and output vectors, so the probability function of constraints is defined as follows:

$$
\begin{aligned}
&\mathrm{Min}\theta \\
&\mathrm{S.t.} \\
&P\left\{\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}\tilde{x}_{ij} \le \theta \sum_{j=1}^{n}\tilde{x}_{ij}\right\} \ge 1-\alpha, \quad i=1,\dots,m, \\
&P\left\{\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}\tilde{y}_{rj} \ge \sum_{j=1}^{n}\tilde{y}_{rj}\right\} \ge 1-\alpha, \quad r=1,\dots,s, \\
&\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk} = 1, \quad k=1,\dots,n, \\
&\lambda_{jk} \ge 0, \quad j,k=1,\dots,n.
\end{aligned} \tag{1}
$$

By calculating the normal distribution, the constraints of *model (1)* are defined as *model (2)* as follows [15]:

$$
\begin{aligned}
&\mathrm{Min}\theta \\
&\mathrm{S.t.} \\
&\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}\tilde{x}_{ij} + s_i^- - \varphi^{-1}(\alpha)\times u_i(\lambda,\theta) = \theta\sum_{j=1}^{n}\tilde{x}_{ij}, \quad i=1,\dots,m, \\
&\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk}\tilde{y}_{rj} + s_r^+ - \varphi^{-1}(\alpha)\times u_r(\lambda) = \sum_{j=1}^{n}\tilde{y}_{rj}, \quad r=1,\dots,s, \\
&v_i^2(\lambda,\theta) = \sum_{j=1}^{n}\sum_{l=1}^{n}(\left[\sum_{k=1}^{n}\lambda_{jk}\sum_{k=1}^{n}\lambda_{jk}\right]\mathrm{cov}(\tilde{x}_{ij},\tilde{x}_{il}) \\
&+\theta^2\sum_{j=1}^{n}\sum_{l=1}^{n}\mathrm{cov}(\tilde{x}_{ij},\tilde{x}_{il}) - 2\theta\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{n}\lambda_{jk}\mathrm{cov}(\tilde{x}_{ij},\tilde{x}_{il}), \\
&u_r^2(\lambda) = \sum_{j=1}^{n}\sum_{l=1}^{n}(\left[\sum_{k=1}^{n}\lambda_{jk}\sum_{k=1}^{n}\lambda_{jk}\right]\mathrm{cov}(\tilde{y}_{rj},\tilde{y}_{rl}), \\
&+\sum_{j=1}^{n}\sum_{l=1}^{n}\mathrm{cov}(\tilde{y}_{rj},\tilde{y}_{rl}) - 2\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{n}\lambda_{jk}\mathrm{cov}(\tilde{y}_{rj},\tilde{y}_{rl}), \\
&\sum_{j=1}^{n}\sum_{k=1}^{n}\lambda_{jk} = 1, \quad k=1,\dots,n, \\
&\lambda_{jk} \ge 0, \quad j,k=1,\dots,n.
\end{aligned} \tag{2}
$$

Also, the projection of inputs and outputs is as follows:

$$
\begin{aligned}
&\hat{x}_{ik}(\alpha) = \sum_{j=1}^{n}\lambda_{jk}^{*}\hat{x}_{ij}, \quad i=1,\dots,m. \\
&\hat{y}_{rk}(\alpha) = \sum_{j=1}^{n}\lambda_{jk}^{*}\hat{y}_{rj}, \quad r=1,\dots,s.
\end{aligned} \tag{3}
$$

Mozaffari & Ostovan | Big. Data. Comp. Vis. 1(3) (2021) 146-155

2.2 | DEA-R Input Oriented

The DEA-R envelopment model is proposed as a *model (4)* in input-oriented, assuming input-to-output ratios are defined [29].

$$
\begin{aligned}
\min \quad & \theta_R \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \left(\frac{x_{ij}}{y_{rj}}\right) \leq \theta \left(\frac{x_{io}}{y_{ro}}\right) \quad i=1,\dots,m,\ \ r=1,\dots,s, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0 \qquad j=1,\dots,n.
\end{aligned} \tag{4}
$$

Solve the linear programming problem to minimize *model (4)* the amount of performance represented by θ_R^*. Where $0 < \theta_R^* \leq 1$.

3 | Finding Projection of a Two-Stage Supply Chain with Random Data, in DEA-R

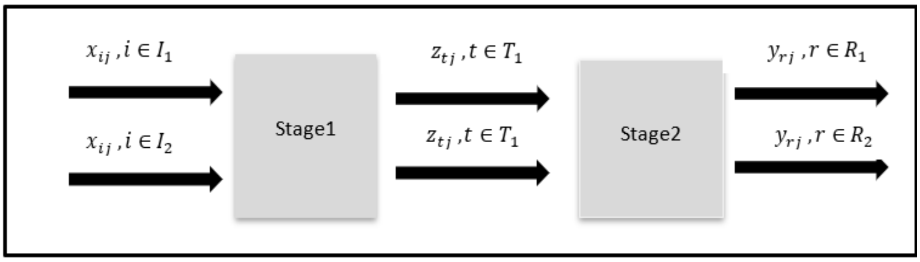

Fig. 1. Two-stage supply chain by considering sustainability factors.

According to *Fig. 1*, suppose there is a two-stage supply chain in which the inputs are classified into two categories I_1 and I_2 and the input vectors into two categories T_1 and T_2, $I = \{1, \dots, m\} = I_1 \cup I_2$ and $T = \{1, \dots, h\} = T_1 \cup T_2$ and the output vectors include two categories $R = \{1, \dots, s\} = R_1 \cup R_2$.

In the first stage, considering that the input vectors as $\tilde{x}_j = (\tilde{x}_{1j}, \tilde{x}_{2j}, \dots, \tilde{x}_{mj})^T$ and the output vectors as $\tilde{z}_j = (\tilde{z}_{1j}, \tilde{z}_{2j}, \dots, \tilde{z}_{dj})^T$, so in DEA-R in the input-oriented where the ratios $\frac{\tilde{x}_{ij}}{\tilde{z}_{tj}}$ $\forall i, j, t$ are defined, the CRA model in DEA-R in the first step is defined as follows.

$$
\begin{aligned}
& \text{Min}\ w_1 \rho'_1 + w_2 \rho''_1 \\
& \text{S.t.} \\
& \textstyle\sum_{j=1}^{n} \sum_{k=1}^{n} \gamma'_{jk} \frac{\tilde{x}_{ij}}{\tilde{z}_{tj}} + \sigma'^{-}_{it} - \tau'^{-1}(\alpha) \times \vartheta'_{it}(\gamma', \rho'_1) = \rho'_1 \sum_{j=1}^{n} \frac{\tilde{x}_{ij}}{\tilde{z}_{tj}} \quad , i \in I_1, t \in T_1, \\
& \textstyle\sum_{j=1}^{n} \sum_{k=1}^{n} \gamma'_{jk} \frac{\tilde{x}_{ij}}{\tilde{z}_{tj}} + \sigma'^{-}_{it} - \tau'^{-1}(\alpha) \times \vartheta'_{it}(\gamma', \rho''_1) = \rho''_1 \sum_{j=1}^{n} \frac{\tilde{x}_{ij}}{\tilde{z}_{tj}} \quad , i \in I_2, t \in T_2, \\
& \textstyle\sum_{j=1}^{n} \sum_{k=1}^{n} \gamma'_{jk} = 1, \qquad k = 1, \dots, n, \\
& \gamma'_{jk} \geq 0, \qquad j, k = 1, \dots, n,
\end{aligned} \tag{5}
$$

$$\vartheta'_{it}{}^{2}(\gamma',\rho\prime_1)=\sum_{j=1}^{n}\sum_{l=1}^{n}(\left[\sum_{k=1}^{n}\gamma'_{jk}\sum_{k=1}^{n}\gamma'_{jk}\right]\operatorname{cov}\left(\frac{\widetilde{x_{ij}}}{\widetilde{z_{tj}}},\frac{\widetilde{x_{il}}}{\widetilde{z_{tl}}}\right),$$
$$+\rho\prime_1{}^{2}\sum_{j=1}^{n}\sum_{l=1}^{n}\operatorname{cov}\left(\frac{\widetilde{x_{ij}}}{\widetilde{z_{tj}}},\frac{\widetilde{x_{il}}}{\widetilde{z_{tl}}}\right)-2\rho\prime_1\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{n}\gamma\prime_{jk}\operatorname{cov}\left(\frac{\widetilde{x_{ij}}}{\widetilde{z_{tj}}},\frac{\widetilde{x_{il}}}{\widetilde{z_{tl}}}\right),i\in I_1,t\in T_1$$

$$\vartheta'_{it}{}^{2}(\gamma',\rho\prime_1)=\sum_{j=1}^{n}\sum_{l=1}^{n}(\left[\sum_{k=1}^{n}\gamma'_{jk}\sum_{k=1}^{n}\gamma'_{jk}\right]\operatorname{cov}\left(\frac{\widetilde{x_{ij}}}{\widetilde{z_{tj}}},\frac{\widetilde{x_{il}}}{\widetilde{z_{tl}}}\right),$$
$$+\rho\prime\prime_1{}^{2}\sum_{j=1}^{n}\sum_{l=1}^{n}\operatorname{cov}\left(\frac{\widetilde{x_{ij}}}{\widetilde{z_{tj}}},\frac{\widetilde{x_{il}}}{\widetilde{z_{tl}}}\right)-2\rho\prime\prime_1\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{n}\gamma\prime_{jk}\operatorname{cov}\left(\frac{\widetilde{x_{ij}}}{\widetilde{z_{tj}}},\frac{\widetilde{x_{il}}}{\widetilde{z_{tl}}}\right),i\in I_2,t\in T_2.$$

In which the projection of the ratio of inputs to outputs of the first stage is as follows.

$$\frac{\widehat{x}_{ik}(\alpha)}{\widehat{z}_{tk}(\alpha)}=\sum_{j=1}^{n}\gamma'^{*}_{jk}\frac{\widetilde{x}_{ij}}{\widetilde{z}_{tj}}\qquad ,i\in I_1,t\in T_1,$$
$$\frac{\widehat{x}_{ik}(\alpha)}{\widehat{z}_{tk}(\alpha)}=\sum_{j=1}^{n}\gamma'^{*}_{jk}\frac{\widetilde{x}_{ij}}{\widetilde{z}_{tj}}\qquad ,i\in I_2,t\in T_2\ . \tag{6}$$

In the second stage, the vectors $\tilde{z}_j=(\tilde{z}_{1j},\tilde{z}_{2j},\dots,\tilde{z}_{dj})^T$ are the input vectors and $\tilde{y}_j=(\tilde{y}_{1j},\tilde{y}_{2j},\dots,\tilde{y}_{sj})^T$ are the output vectors to be. Therefore, by defining the ratio $\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}}$ $\forall r,j,t$ the central resource allocation model in the second stage in DEA-R is proposed as follows.

$$\text{Min}\ w_3\rho\prime_2+w_4\rho''_2$$
$$\text{S.t.}$$
$$\sum_{j=1}^{n}\sum_{k=1}^{n}\gamma''_{jk}\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}}+\sigma''^{-}_{tr}-\tau''^{-1}(\alpha)\times\vartheta''_{tr}(\gamma'',\rho\prime_2)=\rho\prime_2\sum_{j=1}^{n}\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}}\qquad ,t\in T_1,r\in R_1,$$
$$\sum_{j=1}^{n}\sum_{k=1}^{n}\gamma''_{jk}\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}}+\sigma''^{-}_{tr}-\tau''^{-1}(\alpha)\times\vartheta''_{tr}(\gamma'',\rho\prime\prime_2)=\rho\prime\prime_2\sum_{j=1}^{n}\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}}\qquad ,t\in T_2,r\in R_2,$$
$$\sum_{j=1}^{n}\sum_{k=1}^{n}\mu\prime\prime_{jk}=1,\qquad k=1,\dots,n,$$
$$\mu\prime\prime_{jk}\geq 0,\qquad j,k=1,\dots,n, \tag{7}$$

$$\vartheta''_{tr}{}^{2}(\gamma'',\rho\prime_2)=\sum_{j=1}^{n}\sum_{l=1}^{n}(\left[\sum_{k=1}^{n}\gamma''_{jk}\sum_{k=1}^{n}\gamma''_{jk}\right]\operatorname{cov}\left(\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}},\frac{\widetilde{z_{tl}}}{\widetilde{y_{rl}}}\right),$$
$$+\rho\prime_2{}^{2}\sum_{j=1}^{n}\sum_{l=1}^{n}\operatorname{cov}\left(\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}},\frac{\widetilde{z_{tl}}}{\widetilde{y_{rl}}}\right)-2\rho\prime_2\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{n}\gamma\prime_{jk}\operatorname{cov}\left(\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}},\frac{\widetilde{z_{tl}}}{\widetilde{y_{rl}}}\right),t\in T_1,r\in R_1,$$

$$\vartheta''_{tr}{}^{2}(\gamma'',\rho\prime\prime_2)=\sum_{j=1}^{n}\sum_{l=1}^{n}(\left[\sum_{k=1}^{n}\gamma''_{jk}\sum_{k=1}^{n}\gamma''_{jk}\right]\operatorname{cov}\left(\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}},\frac{\widetilde{z_{tl}}}{\widetilde{y_{rl}}}\right),$$
$$+\rho\prime\prime_2\sum_{j=1}^{n}\sum_{l=1}^{n}\operatorname{cov}\left(\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}},\frac{\widetilde{z_{tl}}}{\widetilde{y_{rl}}}\right)-2\rho\prime\prime_2\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{n}\gamma\prime_{jk}\operatorname{cov}\left(\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}},\frac{\widetilde{z_{tl}}}{\widetilde{y_{rl}}}\right),t\in T_2,r\in R_2,$$

$$\frac{\widehat{z}_{tk}(\alpha)}{\widehat{y}_{rk}(\alpha)}=\sum_{j=1}^{n}\gamma''^{*}_{jk}\frac{\widetilde{z}_{tj}}{\widetilde{y}_{rj}}\qquad ,t\in T_1\ ,r\in R_1,$$
$$\frac{\widehat{z}_{tk}(\alpha)}{\widehat{y}_{rk}(\alpha)}=\sum_{j=1}^{n}\gamma''^{*}_{jk}\frac{\widetilde{z}_{tj}}{\widetilde{y}_{rj}}\qquad ,t\in T_2\ ,\ r\in R_2 \tag{8}$$

Eq. (8) shows the input/output ratios of the second stage. Finally, the overall efficiency model of central resource allocation in DEA-R is proposed as the *model (9)*.

Mozaffari & Ostovan | Big. Data. Comp. Vis. 1(3) (2021) 146-155

$$\mathrm{Min}\{\Delta_1, \Delta_2\}$$

S. t.

$$\sum_{j=1}^{n}\sum_{k=1}^{n}\Upsilon'_{jk}\frac{\widetilde{x_{ij}}}{\widetilde{z_{tj}}}+\sigma'^{-}_{it}-\tau'^{-1}(\alpha)\times\vartheta'_{it}(\Upsilon',\rho)=\Delta_1\sum_{j=1}^{n}\frac{\widetilde{x_{ij}}}{\widetilde{z_{tj}}} \quad ,i\in I_1\cup I_2, t\in T_1\cup T_2,$$

$$\sum_{j=1}^{n}\sum_{k=1}^{n}\Upsilon'_{jk}=1, \qquad k=1,\dots,n,$$

$$\gamma'_{jk}\geq 0, \qquad j,k=1,\dots,n,$$

$$\vartheta'^{2}_{it}(\Upsilon',\rho'_1)=\sum_{j=1}^{n}\sum_{l=1}^{n}(\left[\sum_{k=1}^{n}\Upsilon'_{jk}\sum_{k=1}^{n}\Upsilon'_{jk}\right]\mathrm{cov}\left(\frac{\widetilde{x_{ij}}}{\widetilde{z_{tj}}},\frac{\widetilde{x_{il}}}{\widetilde{z_{tl}}}\right),$$
$$+\Delta_1^{2}\sum_{j=1}^{n}\sum_{l=1}^{n}\mathrm{cov}\left(\frac{\widetilde{x_{ij}}}{\widetilde{z_{tj}}},\frac{\widetilde{x_{il}}}{\widetilde{z_{tl}}}\right)-2\Delta_1\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{n}\Upsilon'_{jk}\,\mathrm{cov}\left(\frac{\widetilde{x_{ij}}}{\widetilde{z_{tj}}},\frac{\widetilde{x_{il}}}{\widetilde{z_{tl}}}\right), i\in I_1\cup I_2, t\in T_1\cup T_2, \tag{9}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{n}\Upsilon''_{jk}\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}}+\sigma''^{-}_{tr}-\tau''^{-1}(\alpha)\times\vartheta''_{tr}(\Upsilon'',\rho)=\Delta_2\sum_{j=1}^{n}\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}} \quad ,t\in T_1\cup T_2, r\in R_1\cup R_2,$$

$$\sum_{j=1}^{n}\sum_{k=1}^{n}\Upsilon''_{jk}=1, \qquad k=1,\dots,n,$$

$$\mu''_{jk}\geq 0, \qquad j,k=1,\dots,n,$$

$$\vartheta''^{2}_{tr}(\Upsilon'',\rho'_2)=\sum_{j=1}^{n}\sum_{l=1}^{n}(\left[\sum_{k=1}^{n}\Upsilon''_{jk}\sum_{k=1}^{n}\Upsilon''_{jk}\right]\mathrm{cov}\left(\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}},\frac{\widetilde{z_{tl}}}{\widetilde{y_{rl}}}\right),$$
$$+\Delta_2\sum_{j=1}^{n}\sum_{l=1}^{n}\mathrm{cov}\left(\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}},\frac{\widetilde{z_{tl}}}{\widetilde{y_{rl}}}\right)-2\Delta_2\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{k=1}^{n}\Upsilon'_{jk}\,\mathrm{cov}\left(\frac{\widetilde{z_{tj}}}{\widetilde{y_{rj}}},\frac{\widetilde{z_{tl}}}{\widetilde{y_{rl}}}\right), t\in T_1\cup T_2, r\in R_1\cup R_2.$$

In *model (9)*, the constraints are considered as a combination of the constraints of *models (5)* and *(7)*. Also, the projection of input to output ratios in overall performance is as follows.

$$\frac{\hat{x}_{ik}(\alpha)}{\hat{z}_{tk}(\alpha)}=\sum_{j=1}^{n}\Upsilon'^{*}_{jk}\frac{\tilde{x}_{ij}}{\tilde{z}_{tj}} \quad ,i\in I_1, t\in T_1,$$

$$\frac{\hat{x}_{ik}(\alpha)}{\hat{z}_{tk}(\alpha)}=\sum_{j=1}^{n}\Upsilon'^{*}_{jk}\frac{\tilde{x}_{ij}}{\tilde{z}_{tj}} \quad ,i\in I_2, t\in T_2,$$

$$\frac{\hat{z}_{tk}(\alpha)}{\hat{y}_{rk}(\alpha)}=\sum_{j=1}^{n}\mu'^{*}_{jk}\frac{\tilde{z}_{tj}}{\tilde{y}_{rj}} \quad ,t\in T_1, r\in R_1, \tag{10}$$

$$\frac{\hat{z}_{tk}(\alpha)}{\hat{y}_{rk}(\alpha)}=\sum_{j=1}^{n}\Upsilon''^{*}_{jk}\frac{\tilde{z}_{tj}}{\tilde{y}_{rj}} \quad ,t\in T_2, r\in R_2.$$

4 | Case Study

Table 1 shows the supply chain data of 11 airlines in Iran during 2017-2011.

Tabe 1. Inputs, intermediates, and outputs in the supply chain.

DMU	Economic Factor X_1	Economic Factor X_2	Social Factor Z_1	Social Factor Z_2	Environmental Factor Y_1	Environmental Factor Y_2
Iran Air	N(0.7897,0.0195)	N(0.8156,0.0220)	N(0.7598,0.0206)	N(0.7174,0.0281)	N(0.7343,0.0515)	N(0.6617,0.0276)
Air Tour	N(0.7847,0.0268)	N(0.5156,0.0504)	N(0.8234,0.0243)	N(0.3327,0.0009)	N(0.6940,0.0561)	N(0.9452,0.0053)
Aseman	N(0.6595,0.0270)	N(0.6024,0.0504)	N(0.8691,0.0239)	N(0.7979,0.0210)	N(0.8655,0.0134)	N(0.8423,0.0196)
Mahan Air	N(0.7130,0.0356)	N(0.8893,0.0061)	N(0.8813,0.0065)	N(0.8630,0.0062)	N(0.8834,0.0256)	N(0.5380,0.0433)
Kish Air	N(0.6533,0.0392)	N(0.5964,0.0978)	N(0.6870,0.0352)	N(0.4044,0.0133)	N(0.7140,0.0766)	N(0.6464,0.0376)
Taban Air	N(0.5428,0.1112)	N(0.5042,0.1415)	N(0.5179,0.0776)	N(0.2948,0.0198)	N(0.5665,0.0981)	N(0.6138,0.0636)
Zagros	N(0.3825,0.1063)	N(0.7247,0.0571)	N(0.5554,0.0612)	N(0.3676,0.0358)	N(0.4866,0.1007)	N(0.5174,0.1311)
Caspian	N(0.4941,0.0766)	N(0.1463,0.1417)	N(0.4798,0.1424)	N(0.2676,0.0452)	N(0.4549,0.0942)	N(0.4083,0.1275)
Naft Air	N(0.7896,0.0111)	N(0.7378,0.0395)	N(0.9013,0102)	N(0.1750.0.0009)	N(0.6970,0.0537)	N(0.6786,0.0575)
Ata Air	N(0.4915,0.1059)	N(0.4620,0.0997)	N(0.5199,0.0562)	N(0.4192.0.0425)	N(0.5857,0.1372)	N(0.7274,0.0595)
Meraj	N(0.5101,0.0774)	N(0.1470,0.1415)	N(0.3565,0.0974)	N(0.0952,0.0022)	N(0.2001,0.1404)	N(0.1498,0.1406)

Table 2 shows the projection ratios of inputs to outputs of the first stage that Ata Air in the ratio of first input to the first output and Naft Air in the ratio of first input to second output and the ratio of the second input to the first output and input ratio The second to the second output has the lowest value.

Table 2. The efficient projection for airlines by solving models (5) and (6).

	DEA-R stage1			
	$\frac{X1}{z1}$	$\frac{X1}{z2}$	$\frac{X2}{z1}$	$\frac{X2}{z2}$
Iran Air	1.0394	1.1008	1.0734	1.1369
Airtour	0.8954	0.9291	1.0332	1.0704
Aseman	0.8091	0.8270	1.0052	1.0272
Mahan Air	0.8426	0.9111	0.8134	0.8815
Kish Air	0.8354	0.9041	0.8036	0.8717
Taban Air	0.8593	0.9275	0.8361	0.9043
Zagros	0.8535	0.9800	0.9739	1.1668
Caspian	0.9562	1.0222	0.9674	1.0362
Naft Air	0.8322	0.9009	0.7993	0.8673
Ata Air	0.6887	1.0405	1.0348	1.9714
Meraj	1.0394	1.1008	1. 0734	1.1369

In *Table 2*, which shows the projection of input-to-output ratios in the second stage, Airtour has the lowest input-to-output ratio, first-to-second-to-second-out, second-to-first-to-output, and second-to-second-to-output ratios. The values in the projection are proportions.

Table 3. The efficient projection for airlines by solving models (7) and (8).

	DEA-R stage2			
	$\frac{z1}{y1}$	$\frac{z1}{y2}$	$\frac{z2}{y1}$	$\frac{z2}{y2}$
Iran Air	0.3423	0.3889	0.1675	0.1845
Airtour	0.1903	0.2061	0.1001	0.1063
Aseman	0.3326	0.3269	0.2108	0.1988
Mahan Air	0.3914	0.3549	0.1875	0.1714
Kish Air	0.3144	0.2983	0.1564	0.1485
Taban Air	0.3144	0.2983	0.1564	0.1485
Zagros	0.3144	0.2983	0.1564	0.1485
Caspian	0.3144	0.2983	0.1564	0.1485
Naft Air	0.3144	0.2983	0.1564	0.1485
Ata Air	0.3144	0.2983	0.1564	0.1485
Meraj	0.3144	0.2983	0.1564	0.1485

In general, according to *Table 4* in the ratio of $\frac{x_1}{z_1}$ of Kish Air Company in the ratio of $\frac{x_1}{z_2}$ of Aseman Company, in the ratio of $\frac{x_2}{z_1}\frac{x_2}{z_2}$, Taban Air Company, in the ratio of $\frac{z_1}{y_1}$, $\frac{z_1}{y_2}$, $\frac{z_2}{y_1}$, $\frac{z_2}{y_2}$ Iran Air, Naft Air, ATA Air, and MERAJ Companies have obtained the lowest values of the aspect ratios.

Table 4. The efficient projection for airlines by solving Eqs. (9) and (10).

	DEA-R Overall							
	$\frac{X1}{z1}$	$\frac{X1}{z2}$	$\frac{X2}{z1}$	$\frac{X2}{z2}$	$\frac{z1}{y1}$	$\frac{z1}{y2}$	$\frac{z2}{y1}$	$\frac{z2}{y2}$
Iran Air	1.0394	1.1008	1.0734	1.1369	0.0225	0.0175	0.0105	0.0083
Airtour	0.8730	0.9291	0.8935	0.9485	0.1852	0.1980	0.1641	0.1787
Aseman	0.7945	0.8466	1.0182	1.1034	0.1852	0.1980	0.1641	0.1787
Mahan Air	0.7681	0.9644	1.0598	1.3755	0.1852	0.1980	0.1641	0.1787
Kish Air	0.7653	0.9910	1.0466	1.3861	0.1852	0.1980	0.1641	0.1787
Taban Air	0.8511	1.1821	0.5705	0.7051	0.1852	0.1980	0.1641	0.1787
Zagros	0.8711	0.9363	0.8453	0.9078	0.1861	0.1987	0.1645	0.1790
Caspian	0.8918	0.9557	0.8776	0.9395	0.1852	0.1980	0.1641	0.1787
Naft Air	0.8833	0.9482	0.8619	0.9244	0.0225	0.0175	0.0105	0.0083
Ata Air	0.9560	1.0193	0.9604	1.0234	0.0225	0.0175	0.0105	0.0083
Meraj	0.9542	1.0275	1.0380	1.1319	0.0225	0.0175	0.0105	0.0083

5 | Conclusion

A supply chain is a process that connects the organization's suppliers, producers, and consumers, and unlike classical data envelopment analysis models, which consider only their inputs and outputs to evaluate decision-making units. Intermediate elements also play an important role in evaluating the supply chain. In this paper, the CRA model is used to evaluate the two-stage supply chain in DEA-R, where the projection of the units is shown on the efficient frontier. The supply chain of eleven airlines in Iran has been studied, some of which are uncontrollable factors, so managers are more exposed to random data and their values in the real world are not known exactly.

Therefore, the model of CRA with random data can be a good criterion for finding the projection of decision-making units on the efficiency frontier.

For future research, the use of non-radial DEA-R models with random data is proposed to calculate the super efficiency and rankings of airlines.

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Mozaffari & Ostovan| Big. Data. Comp. Vis. 1(3) (2021) 146-155

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